SBT Soft Fault Diagnosis in Analog Electronic Circuits: A Sensitivity-Based Approach by Randomized Algorithms

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***Abstract*—This paper addresses the fault diagnosis issue based on a simulation before test philosophy in analog electronic circuits. Diagnosis, obtained by comparing signatures measured at the test nodes with those contained in a fault dictionary, allows for sub-systems testing and fault isolation within the circuit. A novel method for constructing the fault dictionary under the single faulty component/unit hypothesis is proposed. The method, based on a harmonic analysis, allows for selecting the most suitable test input stimuli and nodes by means of a global sensitivity approach efficiently carried out by randomized algorithms. Applicability of the method to a wide class of circuits and its integration in diagnosis tools are granted since randomized algorithms assure that the selection problem can be effectively carried out with a poly-time algorithm independently from the fault space, structure, and complexity of the circuit.**

***Index Terms*—Analog circuits, fault diagnosis, neural classifiers, radial basis function networks, randomized algorithms (RAs), sensitivity analysis.**

I. Introduction

TESTING and diagnosis of electronic devices are fundamental topics in the development and maintenance of safe and reliable complex systems. In both cases, the attention is focused on the detection of faults affecting a subsystem whose appearance generally impairs the global system safety and performance [1].

In a complete fault diagnosis procedure, fault detection and isolation must be carried out together; the effectiveness of the procedure depends on fault detection and isolation performance as well as the complexity of the test phase. While there are established techniques to obtain an automatic diagnosis for a digital circuit, the development of an effective automated diagnosis tool for analog or mixed circuits is still an open research field.

Two major issues make the analysis particularly difficult: the complex nature of the fault mechanism, namely the physical/chemical process leading to a failure, and the unknown values for the actual component parameters (which differ from the nominal values). Parameter deviations depend on the intrinsic nature of the production process of the component and on-the-field deviations such as those related to aging or thermal effects. Such situations, which do not change the circuit topology, are commonly defined as "soft" or parametric faults and may lead to unpredictable incorrect operations depending on their impact on the circuit performance.

Two different approaches are used to obtain a circuit diagnosis in the analog system [2]. The first approach is named simulation after test (SAT) [3]–[5]: fault isolation is obtained by estimating the circuit parameters from the measured circuit outputs. The identification of the circuit parameters is based on the assumption that enough information is available in the measurements and that measurements are mutually independent. The method suffers from several drawbacks associated with the identification procedure if the system is nonlinear or local minima issues arise. In addition, SAT techniques are generally time consuming when applied to large circuits. The alternative approach, that has given satisfactory results in circuit diagnosis, is named simulation before test (SBT) [6]–[10] and ensures a reduced test time also when complex circuits are envisaged.

The SBT approach is based on the comparison of the circuit responses associated with predefined test stimuli with those induced by different fault conditions. A subsequent classification must be considered to solve the fault detection and isolation problems.

Signatures associated with faults are generated during a simulation of the circuit before the diagnosis phase and are collected in a "fault dictionary." The design of a diagnosis SBT-based approach for a circuit under test (CUT) is an articulated process that requires the following:

1) identification of the most controllable and observable nodes in the CUT (test nodes);
2) input stimuli selection, i.e., identification of the most appropriate test stimuli able to excite the CUT so that the faulty-induced effect propagates to an observable node;
3) definition of a circuit signature, i.e., extraction of a set of features from the signals measured at the test nodes. The selected features must be able to highlight faults;
4) definition and construction of a fault dictionary. The elements to be stored are circuit signatures, each of which is labeled by the indication of the faulty unit. The dictionary contains information associated with fault-free and faulty situations;
5) design of the classifier and classification of the actual circuit signature by exploiting the knowledge present in the fault dictionary. The classifier provides the faulty/fault-free indication and, when a fault is identified, its location.

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Due to the continuous nature of the soft fault mechanism and the presence of different sources of noise, a complete "fault dictionary" containing all feasible fault examples cannot obviously be generated. The problem can be partly solved by sampling the fault space and considering an "intelligent" diagnosis system able to generalize from the finite set of fault examples or using efficient pattern recognition techniques [11]. Neural classifiers trained by the fault dictionary have been shown to be promising solutions [6], [7], [12]–[14]. Here, we consider the neural network approach presented by the authors in [14], where input stimuli are sinusoidal waves characterized by different frequencies exciting the CUT at different instants of time. Other solutions based on the wide band stimuli approach, such as those based on white noise [6], are more efficient in terms of analysis time but provide a reduced signal-to-noise ratio.

The circuit signatures constituting the fault dictionary are obtained during simulation by injecting feasible faults in the CUT and measuring the amplitude of the CUT transfer function at different test points. The construction of the fault dictionary is a fundamental topic in the SBT [16], [17] and its optimization represents a critical issue in terms of number and efficacy of the signatures to be inserted.

In this paper, we present a novel *global sensitivity analysis* for the CUT which allows improving available diagnosis tools in the

- selection of the most suitable test points;
- identification of the most effective input stimuli (frequencies).

The suggested method is based on randomized algorithms (RAs) [18]–[21] that allow for removing all hypotheses assumed by the sensitivity related literature such as the small perturbation hypothesis and are characterized by a poly-time complexity independent from the dimension of the perturbation space.

The use of a global sensitivity analysis, independent from any limiting hypotheses about the nature of the CUT and the performance loss function, can be effectively inserted in any SBT methodology and is applicable to a large class of analog circuits.

The structure of the paper is as follows. Section II introduces RAs and the neural classification scheme suggested in [14] as the reference diagnosis system for performance comparison. Section III provides the general diagnosis approach based on RAs and suggests the new method for selecting the most relevant test input stimuli and nodes. Experimental results applied to two CUTs are given in Section IV to show the effectiveness of the proposed method.

II. RANDOMIZED ALGORITHMS AND THE REFERENCE DIAGNOSIS SYSTEM

This section provides the background on RAs needed in subsequent sections as well as the basic description of the neural-based classification scheme considered for fault detection and location.

To grant applicability of the proposed method to a large class of circuits, we have to relax all the hypotheses assumed in the sensitivity/robustness literature; this can be obtained by resorting to RAs [18]–[21].

RAs are sample-based techniques that inherit and integrate the Monte Carlo method and the theory of learning philosophy. Extensive evidence for the effectiveness of such statistical approaches can be found in the control theory community where a great effort has been devoted to the analysis and design of robust controllers [21]–[23].

A. RAs: the Basic Concepts

Denote by y a generic function and by $y(\Delta_i)$ the function affected in some perturbation injection points by the perturbation vector Δ_i. Δ_i belongs to a continuous k-dimensional perturbation space D drawn according to the probability density function pdf_D (*a priori* there is a different pdf for each component of the perturbation vector). At this level, nothing is said about nature and placement of perturbations.

The effect of perturbations affecting y and the discrepancy between y and $y(\Delta_i)$ can be measured according to a discrepancy loss function $u(\Delta)$ which is assumed to be measurable according to Lebesgue with respect to the perturbation space D. Practically, all circuits and significant loss functions are measurable according to Lebesgue.

To compute the sensitivity of a generic function y subject to perturbations, Δs spanning their dominion D according to a given loss function $u(\Delta)$, we have to compute the minimum value γ for which the $u(\Delta) \le \gamma, \forall \Delta \in D$ inequality holds.

A dual probabilistic problem can be considered, which tests whether the deterministic problem $u(\Delta) \le \gamma, \forall\Delta \in D$ holds for a given level of probability or not. In particular, when $\Pr(u(\Delta) \le \gamma) = 1, \forall\Delta \in D$, we have that the deterministic and the probabilistic problems coincide at least on the perturbation dominion $D - \Omega$, where the Lebesgue measure of Ω is null. Denote by $P_\gamma = \Pr(u(\Delta) \le \gamma)$ the probability that the loss function is below a given, but arbitrary, positive value γ, $\forall\Delta \in D$.

In other terms, P_γ represents the volume of the perturbation points satisfying the inequality and hence it provides a measure of the impact of perturbations on $u(\Delta)$. We observe that the probability P_γ is unknown but can be estimated by random sampling of the perturbation space. Therefore, extract N independent and identically distributed Δ_i from D according to pdf_D and generate the indicator function defined as

$$I(\Delta_i) = \begin{cases} 1, & \text{if } u(\Delta_i) \le \gamma \\ 0, & \text{if } u(\Delta_i) > \gamma. \end{cases} \tag{1}$$

An estimate of P_γ can be obtained as

$$\hat{P}_\gamma = \frac{1}{N}\sum_{i=1}^{N} I(\Delta_i). \tag{2}$$

Of course, the adherence of $\hat{P}_\gamma$ to P_γ depends on some required accuracy level ε (i.e., we require that $|\hat{P}_\gamma - P_\gamma| \le \varepsilon$), which, in turn, depends on the number of samples N drawn from D. $\hat{P}_\gamma$ is a random variable depending on the particular extraction of the N samples. In fact, we would have obtained a different estimate $\hat{P}_\gamma$ for each different set of cardinality N drawn from D. To remove this statistical fluctuation, we introduce a confidence

TABLE I
METHOD COMPARISON

METHOD	THE ALGORITHM COMPLEXITY	NUMERICAL COMPLEXITY	LIMITS/ ADVANTAGES
Adjoint Networks[26,27]	MEDIUM/HIGH It requires construction of the Adjoint Network.	MEDIUM Requires the solution of two complex linear systems for each frequency of interest	The small perturbation hypothesis is assumed to transform integral perturbations into differential ones.
Sensitivity (DERIVATIVE)[26]	MEDIUM It requires symbolic calculations and a known transfer function.	LOW / MEDIUM The complexity depends on the transfer function if performed in software.	It is required to have a linear CUT and the small perturbation hypothesis.
Sensitivity (INCREMENTAL) [28,29]	MEDIUM Similar to Sensitivity DERIVATIVE	LOW / MEDIUM The complexity depends on the transfer function if performed in software.	It is required to have a linear CUT. The amplitude perturbations must be carefully selected.
Statistical (RAs/MONTE CARLO METHOD)	LOW (if it uses commercial simulation software)	MEDIUM/HIGH The complexity depends on the required accuracy (and hence, on the CUT complexity).	No limiting assumptions.
Symbolic Network Function Approach [26]	MEDIUM It requires a known transfer function. It is a symbolic method that avoids differential computation.	LOW It requires the transfer function, and it is applicable only when the number of terms is not excessive.	It is required to have a linear CUT and the small perturbation hypothesis.

level $1-\delta$ and we require that the $|\hat{P}_\gamma - P_\gamma| \leq \varepsilon$ inequality holds at least with probability $1-\delta$.

By selecting N as suggested by Chernoff [24]

$$N = \frac{\log \frac{2}{\delta}}{2\varepsilon^2} \tag{3}$$

we grant that

$$\Pr\left\{\left|\hat{P}_\gamma - \Pr\left(u(\Delta) \leq \gamma\right)\right| \leq \varepsilon\right\} \geq 1-\delta, \forall \Delta \in D \tag{4}$$

holds with arbitrary confidence $1-\delta$ and accuracy ε.

Construction of P_γ will be the key point for the subsequent diagnosis approach.

B. Neural-Based Diagnosis System

For completeness, we briefly present the classifier needed to generate a complete diagnosis system. The chosen classifier has been suggested by the authors in [14] and is composed of a radial basis function neural network [15]. The three-layered neural network has Gaussian radial basis units in the hidden layer and linear outputs. The input layer receives the CUT actual fault signature and classifies it as faulty or fault-free by indicating, in the faulty case, the faulty unit. A winner-takes-all philosophy has been considered in the output layer.

The network is trained by data contained in the fault dictionary in three separate steps [21].

- The centers of the hidden node activation functions are placed on the centroids of fault dictionary data clusters. The clustering algorithm used in this work is the Fuzzy C-means.
- The width of the activation function is set by a p-nearest neighbor heuristic.
- The weights of the output linear nodes are found in a supervised way by least square method.

The trained network can be used to diagnose circuits belonging to the envisaged CUT family.

III. RAS-BASED METHODOLOGY TO DESIGN THE FAULT DICTIONARY

Identification of the most effective testing nodes and input stimuli selection are critical points to design an effective fault dictionary and to carry out the circuit diagnosis in a reasonable time.

The designer goals are minimization of the number of test points (each of which requires a measurement), features to be extracted (which impact on the computational load required by

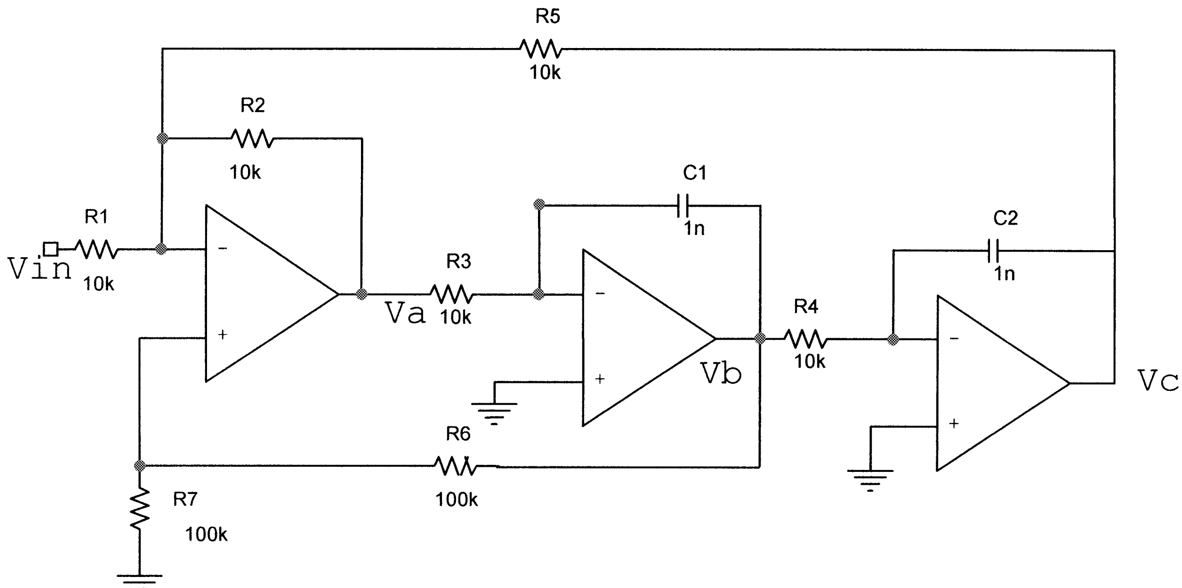


Fig. 1. CUT: a Biquad filter. The nominal values are in the appropriate units.

the feature extraction algorithm), and input stimuli (there is a cost for each input presentation).

A sensitivity analysis can be envisaged to measure the ability of an input stimulus to excite a fault and propagate its effect at a testing point; this ability depends on the input stimulus, the placement and nature of the fault, the structure of the circuit, and the chosen test node. By studying the sensitivity of a test node subject to sinusoidal inputs (at different frequencies) with respect to soft faults, we can identify the most active frequencies and effective test nodes. A sensitivity-based approach can, therefore, be suggested to select the test points and stimuli frequencies.

In general, standard sensitivity analysis methods operate in the small by assuming unreasonable hypotheses about the structure of the circuit and the nature of the fault. The most common assumptions are differentiability for the CUT (which allows for subsequent linearization) and the small faults (deviation) hypothesis. An additional problem to the sensitivity/robustness analysis is posed by the continuity nature of the fault/deviation space, which makes a point-to-point exploration unfeasible. To grant applicability of the proposed methodology to a large class of circuits, we have to relax all the hypotheses assumed in the sensitivity/robustness literature by resorting to RAs.

The main result is that an estimate of the sensitivity degree for a large family of functions once affected by perturbations can be obtained with a poly-time complexity and arbitrary accuracy and confidence regardless of the dimension of the perturbation space (e.g., the number of perturbation injection points). In other words, we can estimate with a limited computational effort the impact of soft faults/deviations on the circuit test points regardless of the circuit complexity.

A methodology for selecting the optimal test frequencies and the optimal test points can be developed by using RAs to compute the sensitivity of y (CUT response) in the candidate test points once the CUT is affected by perturbations in the circuit parameters defined in domain D.

In particular, y describes the function implemented by the ideal CUT, namely the analog circuit in which all components of the circuit (e.g., resistors and capacitors) are characterized by their nominal values. Conversely, a perturbed function $y(\Delta_i)$ models a real CUT in which the parameters differ from the nominal values due to the production process, parameter deviation, soft faults, etc.

Given a controllable node X_i, the first steps of the procedure can be detailed as the following:

- For each candidate test node X_k, denote by $u_k(\Delta_i)$ the variation in amplitude of the transfer function between nodes X_k and X_i at a given frequency f.
- Generate N i.i.d. samples according to the chosen δ and ε. Each deviation Δ_i must be extracted according to the pdf_D that reflects the parameter variation of the CUT associated with the component production. For instance, if the production process generates parameters ruled by a Gaussian distribution centered in their nominal values with tolerance T, the pdf_D of D is Gaussian with standard deviation $T/3$.
- Evaluate the estimate $\hat{P}'_\gamma = \hat{P}_\gamma(\gamma), \forall \gamma \geq 0$ for each candidate frequency f. The $\hat{P}'_\gamma$ curve describes the sensitivity of the output of the CUT subject to the parameter fluctuations in D at the given frequency f.

The $\hat{P}'_\gamma$ curve fully characterizes the behavior of the circuit resulting from the production process. Note that we have not considered a single circuit but an ensemble of circuits; the ensemble contains all those acceptable circuits we could have generated by acquiring the parameters and assembling them according to tolerance T.

We tested that, in several CUTs, a good estimate for the $P_\gamma = P_\gamma(\gamma)$ curve can be obtained by significantly reducing the confidence and accuracy degrees and hence the number of samples N to be extracted. This happens when the $u(\Delta)$ function applied to the specific CUT is reasonably "smooth" in the domain D.

A soft fault is a fault that changes and biases the ensemble behavior of the ideal circuit. Therefore, to evaluate the sensitivity degree of the ensemble, we have to study the effect of faults. Faults can be defined as those deviations of parameters pushing the circuit out of the feasible ensemble and, as such, are characterized by an abnormal behavior. In other terms, we could imagine that faults are generated by an inaccurate production process characterized by larger tolerances (perturbation domain D).

To study the sensitivity of the circuit ensemble on larger D we have to do the following:

- Assume a fault condition when all circuit parameters related to a faulty component deviate abnormally. We could consider, for instance, a Gaussian distribution centered in the nominal value of the parameters and a larger standard deviation $2T/3$.
- Evaluate for each frequency the estimate $\hat{P}''_\gamma = \hat{P}_\gamma(\gamma), \forall\gamma \geq 0$ with respect to the newly perturbed space.
- The sensitivity of the CUT associated with different perturbation domains can be obtained by evaluating the distance between curves $\hat{P}'_\gamma = \hat{P}_\gamma(\gamma)$ and $\hat{P}''_\gamma = \hat{P}_\gamma(\gamma)$. We verified that an effective measure of the sensitivity of the node at frequency f is $E(|\hat{P}'_\gamma - \hat{P}''_\gamma|)$; the expectation is taken with respect to γ.

The proposed sensitivity analysis is particularly flexible and can be applied to any perturbation domain and CUT, regardless of the perturbation magnitude associated with the faulty parameter. The complexity of the CUT does not affect the number N of simulations required to estimate P_γ but it affects the time required to compute $u(\Delta)$. In particular, when a linear behavior is considered and the CUT can be accurately described by a known transfer function, the computational burden is substantially independent from the CUT complexity. Table I presents a comparison between the different sensitivity analysis methods presented in literature and those based on RAs with respect to algorithm and numerical complexities.

The information obtained by the sensitivity analysis can be exploited to identify the test input stimuli and nodes. It is obvious that a test node is relevant when the average sensitivity value is large. Moreover, a frequency is exciting when it induces a large sensitivity value. It must be observed that a fault can be localized only if the set of output deviations is a signature for the considered fault at the selected test frequencies.

In this paper, we select a test stimuli (and node) and we construct the set of test stimuli (and nodes) by considering those yielding the maximum average difference in the sensitivity curves.

For completeness, the final steps of the methodology require the development of the fault dictionary and the design of the classifier. In particular, it is required to do the following:

- generate the fault dictionary by inducing reasonable faults for each possibly faulty unit;
- extract a signature for each fault;
- design the classifier from the fault dictionary.

The whole diagnosis procedure is applied in the next section to two nontrivial CUTs.

IV. Experimental Results

To present how the methodology can be used to identify the most effective test points and frequencies, we consider two examples where faults at component and sub-system level are considered.

A. Component Fault Diagnosis (Biquad Filter)

The CUT of the considered universal filter (Biquad) is shown in Fig. 1; the filter has been designed to have a cut-off frequency of 15.9 kHz. We assume that all components composing the circuit come from a production process of tolerance $T = 1\%$ and that faults act directly at the component level (each component at a time can be affected by a fault).

We identify three interesting observable nodes in the circuit and, in particular, the voltage V_a of the high pass-filter, the voltage V_b of the band-pass filter, and the output V_c of the low-pass filter. The input stimuli are injected into the controllable node by applying a voltage directly at $V_{\rm in}$. By considering the ambiguity groups (faults affecting them constitute an equivalent class), seven fault classes result for the CUT:

— class 1: R6, R7 faulty;
— class 2: R3, C1 faulty;
— class 3: R4, C2 faulty;
— class 4: R5 faulty;
— class 5: R2 faulty;
— class 6: R1 faulty;
— class 7: no faulty components(fault-free condition).

As suggested by the method presented in Section III, we have to generate the sensitivities of the CUT for each of the three test points.

The $\hat{P}'_\gamma$ and $\hat{P}''_\gamma$ curves should be obtained by considering high accuracy and confidence levels, e.g., $\delta = 0.01$ and $\varepsilon = 0.01$ which would require $N = 26\,500$ samples to be extracted to study the circuit ensemble. As in Section II, in several applications, a smaller number of points provide the same estimates. In this case, we discovered that $N = 3000$ works well (from the theory point of view it corresponds to $\delta = 0.01$ and $\varepsilon = 0.03$). We experimentally observed that excellent results can be obtained also with $N = 300$ and, hence, with a reduced computational complexity.

The $E(|\hat{P}'_\gamma - \hat{P}''_\gamma|)$ for the three testing nodes as a function of the frequencies is given in Fig. 2. We have seen that all faults are detectable. It is important to note that the test frequencies and node selection must be performed by considering a high sensitivity and also selectivity, i.e., at the test frequencies, the sensitivity must have a characteristic shape for each fault. From these observations, we selected V_b. We considered, at the end, seven test frequencies extracted in the neighborhood of the cut-off frequencies.

The next step requires designing the classifier. Fault conditions can be simulated by allowing the component parameters of the faulty sub-system to deviate from their nominal value X_n and span the $[0.1X_n, 2X_n]$ interval; a uniform distribution has been considered for its conservative property [25]. The classifier was trained with the 300 examples per class constituting the fault dictionary. It can be seen in Fig. 3 that for a reasonably

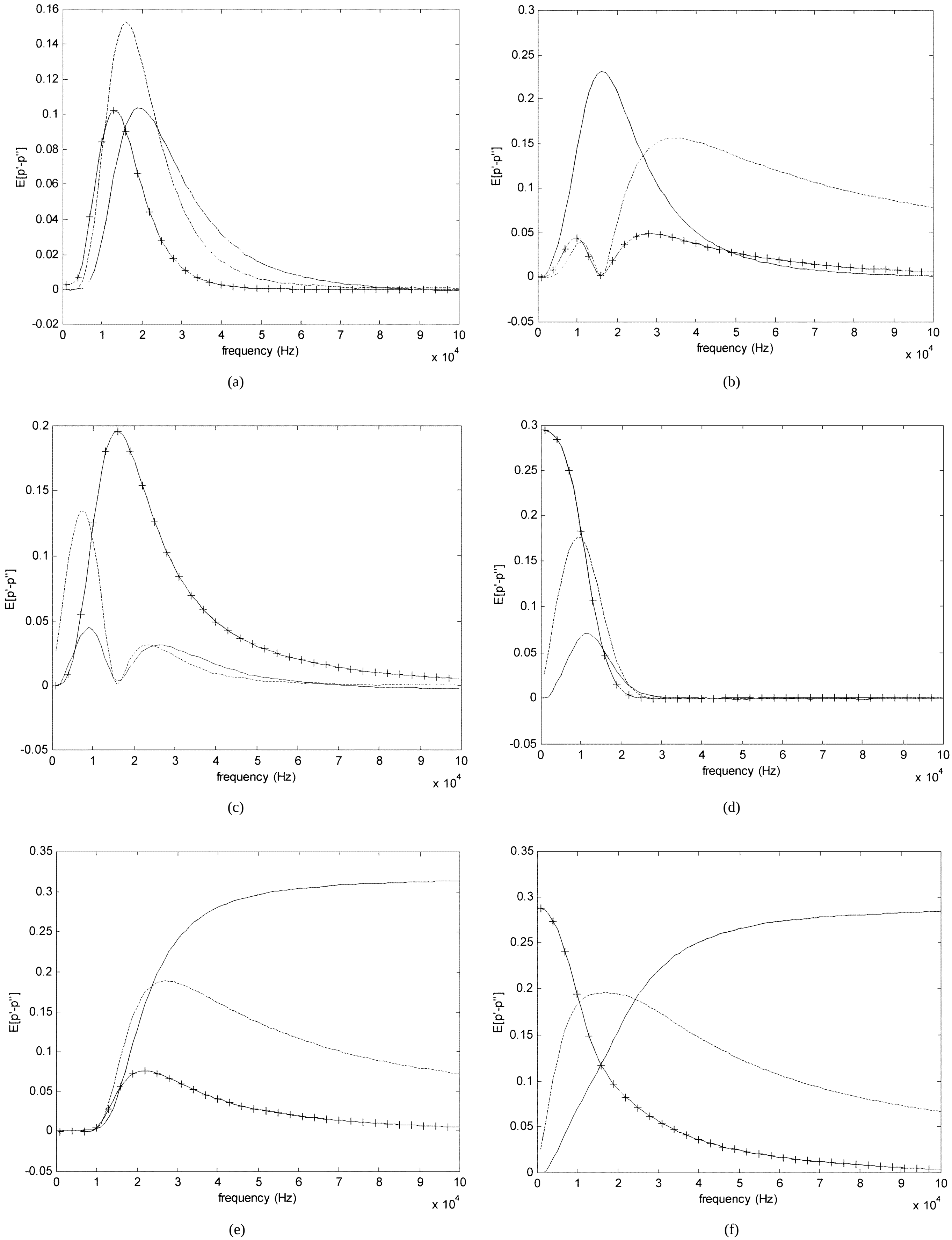


Fig. 2. $E(|\hat{P}'_\gamma - \hat{P}''_\gamma|)$ function for the different fault classes ("-" for node V_a, "- -" for V_b, "+" for node V_c). (a) Class 1: R6, R7 faulty; (b) class 2: R3, C1 faulty; (c) class 3: R4, C2 faulty; (d) Class 4: R5 faulty; (e) class 5: R2 faulty; (f) class 6: R1 faulty.

Fig. 3. Classification error as a function of the hidden node number for the Fig. 2 CUT.

complex topology (larger than 170 hidden nodes), the testing error is below 4%.

B. Sub-System Fault Diagnosis

As a second example, we consider the circuit shown in Fig. 4(a). The CUT consists of four filtering stages and an adder; the transfer function of the CUT is given in Fig. 4(b).

Differently from the previous case we consider faults at the sub-system level instead of at the component level. This is the most common case in complex circuits where the basic element is a sub-system (which, if faulty, can be replaced). The six fault classes listed below are taken into account and, in particular:

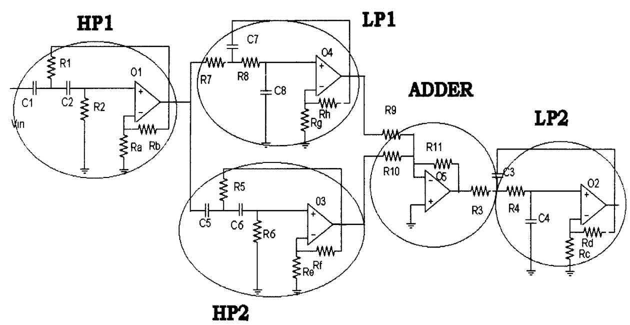
- Class 1: Highpass 1 faulty;
- Class 2: Highpass 2 faulty;
- Class 3: Lowpass 1 faulty;
- Class 4: Lowpass 2 faulty;
- Class 5: Adder faulty;
- Class 6: No faulty units (fault-free condition).

Characterization of the faulty element is related to a loss in performance of some critical feature of the sub-system. In this case, we define a sub-system to be faulty when

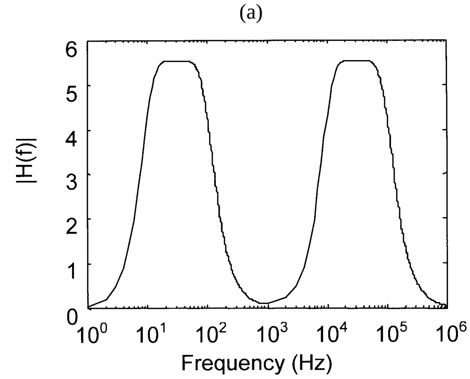
- for each filter stage, the cut-off frequency differs from the nominal value by more than 20%;
- for the adder, the CUT maximum deviation of the CUT response amplitude is larger than 20% with respect to its nominal value.

Fault conditions can be simulated as in the previous example by extracting the component parameters from the $[0.1X_n, 2X_n]$ uniform interval. Only simulations associated with a single faulty unit were inserted in the fault dictionary.

For the considered CUT, the stimulus input is inserted at the input node of the HP1 stage while the test output is that of



(a)



(b)

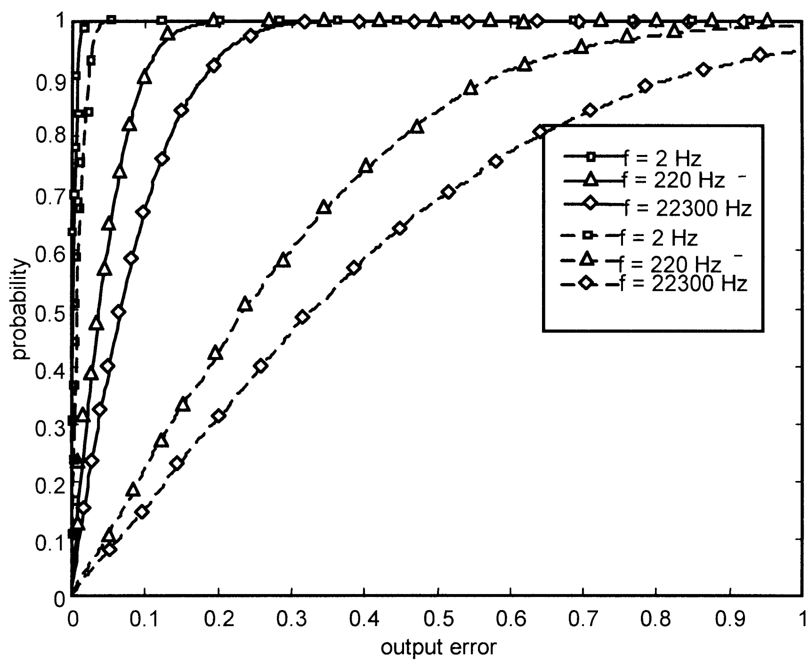
Fig. 4. (a) CUT schematic; (b) CUT transfer function.

LP2. One test point is enough since the filter is composed of cascading noninteracting stages. Moreover, for the structure of the filter, each fault affects the CUT behavior in distinct frequency regions. For this property, the $\hat{P}''_{\gamma}$ estimate can be obtained by sampling all circuit components in their parameter domain simultaneously.

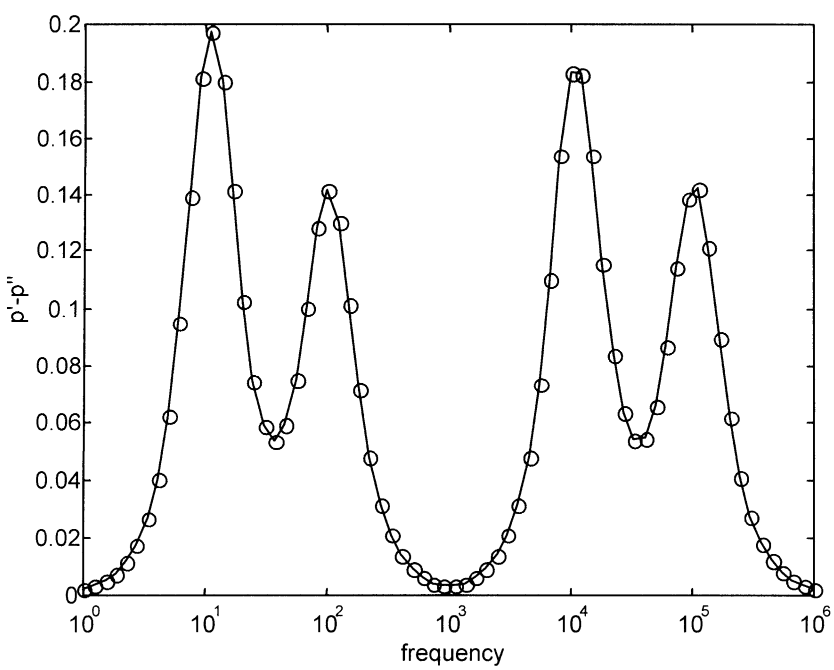
To this end, it is interesting to note that the tolerances of the different components must not necessarily be the same. In the

TABLE II
COMPONENT VALUES FOR THE FIG. 4(A) CUT

HP1	R1=320kΩ ± 10%	R2=320kΩ ±10%	C1=50nF ±5%	C2=50nF ± 5%	Av1=1.75± 2%
HP2	R5= 320 Ω± 10%	R6=320Ω ±10%	C5=50nF ±5%	C6=50nF ± 5%	Av2=1.75 ±2%
LP1	R7=32k Ω ± 10%	R8=32kΩ ±10%	C7=50nF ±5%	C8=50nF ± 5%	Av3=1.75 ±2%
LP2	R3= 32 Ω± 10%	R4=32 Ω±10%	C3=50nF ±5%	C4=50nF ± 5%	Av4=1.75 ±2%
ADDER	R9=1kΩ± 1%	R10=1kΩ ± 1%	R11=1kΩ ± 1%		



(a)



(b)

Fig. 5. (a) $\hat{P}'_\gamma$ continuous line, $\hat{P}''_\gamma$ dashed line; (b) distance between the two curves: $E(|\hat{P}'_\gamma - \hat{P}''_\gamma|)$.

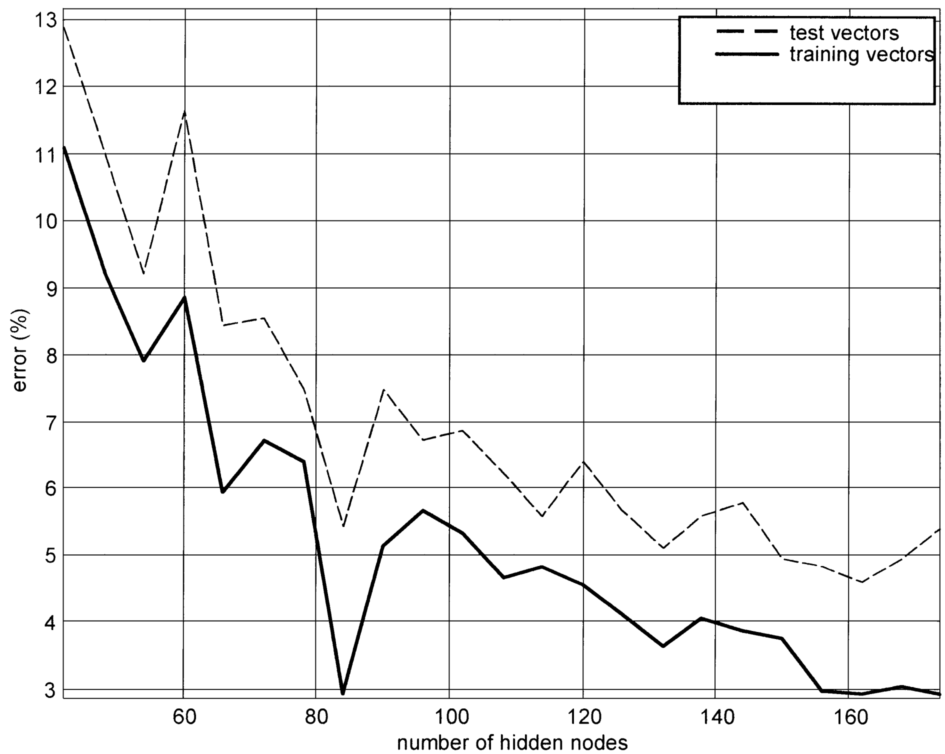


Fig. 6. Classification error as a function of the hidden node number for the CUT in Fig. 5(a).

specific case, the nominal values for parameters and their tolerances are given in Table II.

The sensitivity analysis provided by the method described above led to the results reported in Fig. 5. In particular, Fig. 5(a) shows some examples for $\hat{P}'_\gamma$ and $\hat{P}''_\gamma$ as a function of γ at three different frequencies.

Fig. 5(b) presents the average distance between the curves as a function of the frequency; as discussed before, the frequency plot represents the sensitivity of the output to circuit component variations. Note that there are four frequency regions whose peaks are in the neighborhood of the filter cut-off frequencies. As expected, the sensitivity is a maximum in such points and we have to select the test signal frequencies in these four regions.

We extracted 10 test frequencies from these areas and we further reduced the dimension of the input space to six by means of principal components analysis techniques. The RBF classifiers have been developed based on the fault dictionary that contained a training set of 350 faults per class for each of six features (the ones coming from the PCA).

Classification results are given in Fig. 6, where it is shown that the classification error is a function of the number of the hidden nodes of the classifier. The test error is below 6% with a reasonable complexity for the neural network (85 hidden nodes).

V. Conclusion

In this paper, a novel method for selecting the test input stimuli and nodes for analog circuits is presented. The method is based on a sensitivity analysis carried out with a polynomial time by RAs. The statistical behavior of the circuit output (or of a CUT performance parameter) is evaluated by means of a reasonable number of Monte Carlo simulations (Pspice), independently from the dimension of the component parameter space. Results, obtained by applying the diagnosis technique presented in [14] to an optimized "fault dictionary," have been presented. The classification error improvement in fault detection and isolation with respect to the optimized procedure suggested in [14] is around 2%, which represents a relevant increment due to the cost of the devices to be tested. In addition to the performance improvement, the methodology can be easily and effectively implemented in an automatic tool for circuit diagnosis.

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